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having endeavoured by popular clamour to overthrow a just and necessary tax, and with having argued against it from individual and selfish motives. Neither of these accusations is true. The supporters of the tax, on the other hand, have resisted every attempt to inquire into its injustice, and have brought forward in its favour arguments as transparent as they are feeble. This is not the way in which a great national question ought to be met. Men's minds have expanded within the last fifty years in a ratio previously unknown; and the present generation look for something more than empty declamation, or appeals to a former statesman and his measures. They cannot accept as unerring truths the exploded dogmas of Mr. Pitt, merely because they have been hallowed by the lapse of half a century, or because they have received the sanction of preceding and perhaps subservient senates.

On the Reliability of Data, when tested by the conclusions to which they lead. By EDWIN JAMES FARREN, one of the Vice-Presidents of the Institute of Actuaries.

[Read before the Institute 22nd March, 1853, and ordered by the Council to be printed.]

IT is a well-known feature in the advancement of learning, that the more knowledge becomes prevalent, the less positive do opinions become. Not only in theology does the staggering demand of "What is truth?" assail us, but we meet with the same impregnable question in all the pursuits of literature and science, obliging us to admit that the truth of yesterday is not always the truth of to-day, and that to-morrow may still be about to offer us some third form.

The older class of writers felt no such difficulty. They conceived they were in possession of certain abstract truths or data that might be considered as fundamental axioms, and that by these axioms might all conclusions whatsoever be tried. If the conclusions were incredible, it was the logic that was to be impeached, not the axioms; for it was taken as the reigning axiom of all, that axioms themselves could not be misconceived or defective. The more general lover of research owes it to mathematicians, that this mechanical form of procedure, that this exaltation of one class of assumptions as tyrants over all others, has become disregarded; for it was by labourers in the exact sciences that it was first fully shown, that starting from

presumed axioms, and proceeding step by step with the precisest precision of logical sequence, that not unfrequently one presumed axiom considered as a premiss could be made to actually deny another presumed axiom considered as a conclusion. The inference was thus gradually established, though not without considerable labour of illustration, that the human mind really possesses neither an innate nor an acquired power of distinguishing axioms, and that not absolute truth, but merely the temporal discussion of the varying definitions of truth, is all that our reasoning powers can strictly pretend to.

The recognition of this somewhat unpalatable doctrine, of the mere probability of truth occupying the place of truth itself, has, nevertheless, opened the way for a more tempered style of demonstration and argument, in which not only premisses or data are allowed to shape out conclusions, but general conclusions are not wholly precluded from having a quasi-suggestive power over assumptions or the reliability of data. We have a well-known illustration of this in algebra, in which it occasionally happens that though starting with what seem to be undeniably positive symbols, yet that we gradually arrive at negative conclusions. The elder school of writers, finding that in many of such cases they could not impeach the logic, were content to evade the difficulties by calling them irreducible; but the modern mathematician at once looks upon them as warnings, not merely that he must re-examine his logic, but more especially the nature of his original postulates.

In life assurance, considered as a branch of applied mathematics, we are also to consider we receive similar warnings, whenever we are led to the portrayal of paradoxical conclusions, setting us at variance with our otherwise well ascertained general notions of the character of life and death. An instance is afforded us in the old assumption that the premium for a year's insurance of a healthy man at one age should be neither greater nor less than the premium at another; but by the aid of such a premiss the conclusion could be arrived at, that the mass of healthy men should gradually pass from age to age without showing any of the well-known ravages of time, and that healthy sires should not be distinguishable from their healthy sons. The more such paradoxical conclusions became displayed, the more they re-acted upon the premiss from which they sprung, and finally engendered the now prevailing notion, that the mere circumstance of age, is of itself to be reckoned, if not among the actual diseases of life, yet of so cognate a character as to be capable of creeping on by such insidious degrees as to leave

a man in apparent health all his life, and yet become the ultimate cause of his death.

There must be many members of this Institute who will scarcely believe it possible that there are still writers who do not consider time as an enemy to vitality, but that it acts, not only during early childhood or the period of growth, but even at 40, with a sort of curative power, that is supposed to be only partially afforded to the healthy man of 20, until he shall have doubled both his age and his experience of what our common enemy time can really do. And yet the question of "*Is there a materially greater risk in the assurance of a select life of from 40 to 45 than of a select life from 20 to 25 for one year?*" has been recently and somewhat formally put, as apocryphal, to the present writer as the author of some life contingency tables, in which he has virtually considered, that however firmly a man may appear to be in possession of what is called select health, yet that he is still infected with the growth of the disease called time or age, and that when seeking a year's or a day's, nay even a moment's insurance, should be relatively charged in that respect.

Many of those now present may remember a paper, headed with the above question, being read before this Institute about two years and a half ago; and, from its not having been recommended for printing by the Council, generally thought to have been considerably withdrawn by its ingenious and evidently sincere author, Mr. Spens, of Glasgow. The paper, however, has been recently printed and circulated by its author, who, with a fondly cherished zeal in his own opinions, has further announced that he conceives the experience of certain Scottish Offices will be found confirmatory of his "*favourite theory that in the assurance of a select life for one year I would almost be disposed to give a preference to the advanced age from 40 to 45 over a greatly younger life.*"

It has been already hinted that this is virtually merely a revival of an old notion, and therefore might still be left to constitute the favourite theory of the respected actuary of the Scottish Amicable about half a century after the abandonment of a similar notion by its paternal institution, the Amicable Society of London. But in the present day, when new schemes for life assurance are so eagerly sought for, as to urge the adoption of almost any expedient to attract public attention, Mr. Spens' paper may be inconsiderately made in other hands an authority for also turning aside more material distinctions. Such is the more likely, if the case be allowed to pass wholly unnoticed; because, from the paper having been

originally read and now bearing, at the instance of its author, the inscription of having been read before this Institute, it might seem to be emblematical of the opinions not opposed but current here, and the more especially if not questioned by those who have made the relative insurance of select and diseased life their peculiar study. It has, therefore, been thought useful to bring the subject once more under the consideration of the Institute, that, at least, those fond of discussion may have an opportunity of assenting to or dissenting from Mr. Spens' views, who, in his own earnest desire for truth, is only the more inclined to believe that others are actuated by the same desire, as is shown by the substance of his last paragraph—"That he is sure there is none who will give the subject a more candid consideration than the gentleman to whose work he has alluded."

It will, probably, be considered obvious, from the preceding remarks, that a truly candid consideration of such a subject produces results unfavourable to Mr. Spens' ideas, because sufficient attention does not appear to have been bestowed upon the topic which his own paper has virtually suggested as the best heading for the present one; namely, the reliability of data when tested by the conclusions to which they lead. All who have pondered deeply on this subject, and have no special theory to support, will probably concur in thinking that tabular expositions, whether of English or Scottish data, are more or less trustworthy in exactly the same ratio as they do not ask us to reverse the natural conception: that the general scheme of human existence, like that of other parts of the animated creation, is first to grow and then to decay, and that both the growth and decay are happily so ordained, unless prevented by ignorance and vice, as to pursue the even tenor of their way, moment by moment and year by year, almost imperceptibly until the closing scene.

As individuals, daily observation teaches us that mortality is always at work, because we see deaths occurring at all ages, and such a reflection is obviously not without its moral uses, founded upon a sense of moral equity. But no such claim can be urged for a special hypothesis like the one we are considering, which asks us to suppose that the ratio of mortality is for a long period at a stand-still, merely that it may afterwards hurry forward as if to overtake its neglected victims, and thus to disarrange without pretending to prolong the general span, long since sufficiently defined by the well-known saying of the Psalmist, so excellently rendered in Cranmer's version: "The days of our age are threescore years and ten; and though men be strong that they come to fourscore years, yet is

their strength then but labour and sorrow ; so soon it passes away and we are gone."

Looking at the subject, then, in this light, that life has at least a defined limit, towards which we are obliged to progress as age progresses, it is one of the strongest *primâ facie* arguments of the reliability of data that they do not tend to contravene or weaken so obvious a conclusion. It is in this respect that the totality of the so called "Experience of the London Offices," expressly impugned by Mr. Spens because not confirming his theory, is especially valuable, and has fully justified the expectation of the originators of the experiment, that the reckoning by a mixture of lives and policies would not in such large numbers importantly differ from calculations based upon lives alone. It is, of course, easy to imagine a combination of policies that might produce erratic results ; but the data in question have safely passed through such an ordeal, and, so far from contravening the general laws of mortality derived from a sufficient number of lives alone, have actually, in each and all of the policy-classes, brought a generally concurrent testimony of considerable value, and have thereby given quite as much support to the reliability of the Equitable and London Amicable experiences as has been derived from such experiences themselves. The trustworthiness, therefore, of the combined data in question, though originally a mere tentative experiment, must now be looked upon as an experiment no longer, but as resulting from materials worthy of being placed among the most valuable we possess, whatever may be thought of such future collections as may present themselves.

By undue subdivision of even the most perfect data, a diversity of opinions might be upheld, from an expected mortality of zero to one of cent. per cent. ; and thus the only method (as already more than once hinted) left us, is to judge of the classification and reliability of data by looking to the conclusions to which they may lead us, and thereby avoid the re-presentment of those ancient paradoxes as to the confusion and the reversal of ages, which no statistician of modern times can ever hope to re-establish, however earnest he may be in his judgment, or sincere as to the consummate fitness of his own method of dealing with materials. To pass from youth to age, we must pass from year to year ; and to suppose this progression towards eventual death among healthy persons in general, apart from unhealthy occupation, to be otherwise than gradual as to age, is to attempt to controvert the very argument which the presumption of health is best set up to establish.

In conclusion, it is hoped that in the present paper such a

mixture of courtesy and of dissent, free from personal satire, has prevailed, as may be deemed fitting for the occasion, especially as such elements are of all others the most easy to be departed from.

On the Valuation of Life Contingencies by means of Tables of Single and Joint Lives. By CHARLES JAMES HARGREAVE, Esq., LL.D., F.R.S.

(Extracted by permission from the *Philosophical Magazine*, for Jan. 1853.)

LET $a_1, a_2 \dots a_n$ be the reciprocals of the roots of the expression $1 - s_1x + s_2x^2 - \dots \pm s_{n-1}x^{n-1} \mp s_nx^n$, which call ϕx . We have then $\phi x = (1 - a_1x)(1 - a_2x) \dots (1 - a_nx)$, from which the following series of values may be readily deduced, the Σ implying the sum of all the instances of the form placed under it, so that each expression is a symmetrical function of $a_1, a_2 \dots a_n$.

$$\begin{aligned} \phi(1) &= (1 - a_1)(1 - a_2) \dots (1 - a_n). \text{No. of terms, } 1. \\ -\phi'(1) &= \Sigma a_1(1 - a_2) \dots (1 - a_n). \text{No. of terms, } n. \\ \frac{1}{2}\phi''(1) &= \Sigma a_1a_2(1 - a_3) \dots (1 - a_n). \text{No. of terms, } n \frac{n-1}{2}. \\ -\frac{1}{2.3}\phi'''(1) &= \Sigma a_1a_2a_3(1 - a_4) \dots (1 - a_n). \text{No. of terms, } n \frac{n-1}{2} \frac{n-2}{3}. \\ &\vdots \\ \pm \frac{1}{2.3 \dots p}\phi^{(p)}(1) &= \Sigma a_1a_2a_3 \dots a_p(1 - a_{p+1}) \dots (1 - a_n). \text{No. of terms, } n \frac{n-1}{2} \dots \frac{n-p+1}{p}. \\ &\vdots \\ \pm \frac{1}{2.3 \dots (n-1)}\phi^{(n-1)}(1) &= \Sigma a_1a_2a_3 \dots a_{n-1}(1 - a_n). \text{No. of terms, } n. \\ \mp \frac{1}{2.3 \dots n}\phi^{(n)}(1) &= a_1a_2a_3 \dots a_n. \text{No. of terms, } 1. \end{aligned}$$

The sign is positive when p is even, and negative when p is odd. A single term of the p th expression contains p factors of the form a , and $n-p$ factors of the form $1-a$; and the expression itself is the sum of every term which can be so constructed.

Let there be n persons whose names are $A_1, A_2 \dots A_n$; and let a_p denote the probability that A_p will be living at any given future date. Then by what precedes, we see that the probability that at the given date there will be exactly p out of the set living (implying that exactly $n-p$ in number are dead), is

$$\pm \frac{1}{2.3 \dots p} \phi^{(p)}(1).$$